Math 115

Spring 2019

Lecture 29

$? a^2 + b^2 = c^2 ?$
$y = mx + b$ $?$ $d = rt$

---

① Simplify: $\dfrac{3x}{x^2+9x+14} - \dfrac{6x}{x^2+4x-21}$

$= \dfrac{3x(x-3)}{(x+2)(x+7)(x-3)} - \dfrac{6x(x+2)}{(x-3)(x+7)(x+2)}$

$= \dfrac{3x(x-3) - 6x(x+2)}{(x+2)(x+7)(x-3)} = \dfrac{3x^2-9x-6x^2-12x}{(x+2)(x+7)(x-3)} = \dfrac{-3x^2-21x}{(x+2)(x+7)(x-3)}$

$= \dfrac{-3x(x+7)}{(x+2)(x+7)(x-3)} = \boxed{\dfrac{-3x}{(x+2)(x-3)}}$

② Solve: $x - \dfrac{14}{x-1} = 4 - \dfrac{2x}{x-1}$ LCD $= x-1$
                                                              E.V.: 1

$x(x-1) - 14 = 4(x-1) - 2x$

$x^2-x-14 = 4x-4-2x$

$x^2-x-14 = 2x-4$

$x^2-x-14-2x+4 = 0$

$x^2 - 3x - 10 = 0$

$\rightarrow (x-5)(x+2) = 0$

By Z.F.P.

$x-5 = 0$ $x+2 = 0$

$x = 5$ $x = -2$

$\{-2, 5\}$

③ Simplify: $\dfrac{y - \frac{4}{y}}{y^2 - \frac{8}{y}} = \dfrac{y^2 - 4}{y^3 - 8} = \dfrac{(y+2)(y-2)}{(y-2)(y^2+2y+4)}$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad y^3 - 2^3$

$\quad\quad\quad\quad LCD = y \quad\quad\quad\quad\quad\quad\quad = \boxed{\dfrac{y+2}{y^2+2y+4}}$

④ Solve: $\boxed{\dfrac{3}{x+5}} + \boxed{\dfrac{1}{x-5}} = \boxed{\dfrac{10}{x^2-25}}$ $\quad LCD = (x+5)(x-5)$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ E.V. ± 5

$3(x-5) + 1(x+5) = 10 \quad\longrightarrow\quad x = 5$

$3x - 15 + x + 5 = 10$

$4x - 10 = 10 \quad\quad\quad\quad\quad \varnothing$

$\quad\quad 4x = 20$

⑤ A number plus 5 times its reciprocal is

equal to 6.

Find all such numbers.

Let x be the number

$\quad\quad x + 5 \cdot \dfrac{1}{x} = 6 \quad\quad\quad\quad \longrightarrow x^2 - 6x + 5 = 0$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (x-5)(x-1) = 0$

$\quad\quad LCD = x \quad\quad\quad\quad\quad\quad\quad\quad$ by Z.F.P.

$\quad\quad$ E.V. $0 \quad\quad\quad\quad\quad\quad\quad\quad x-5=0 \quad\quad x-1=0$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad x=5 \quad\quad\quad x=1$

$\quad\quad x^2 + 5 = 6x$

$\quad\quad x^2 + 5 - 6x = 0 \quad\quad\quad\quad\quad\quad\quad \{1, 5\}$

⑥ Find two consecutive odd integers such that the difference of their reciprocals is equal to $\frac{2}{3}$.

$x, x+2$

$\frac{1}{x} - \frac{1}{x+2} = \frac{2}{3}$

→ Solve $\boxed{\frac{1}{x}} - \boxed{\frac{1}{x+2}} = \boxed{\frac{2}{3}}$

$LCD = 3x(x+2)$

$(x+3)(x-1)=0$

By Z.F.P.

$x+3=0$ $x-1=0$

$x=-3$ $x=1$

$\{3$ or

$-3\{-1$

x	x+2
1	3
-3	-1

$3(x+2) - 3x = 2x(x+2)$

$3x + 6 - 3x = 2x^2 + 4x$

$2x^2 + 4x - 6 = 0$

Divide by 2

$x^2 + 2x - 3 = 0$

⑦ Pipe A can fill up an empty pool in 5 hrs.
Pipe B can empty a full pool in 7 hrs.
If the pool is empty, and both pipes are working, how long does it take to fill up the pool?

$$\underset{\substack{\text{Work} \\ \text{by} \\ \text{Pipe A}}}{} - \underset{\substack{\text{Work} \\ \text{by} \\ \text{Pipe B}}}{} = \underset{\substack{\text{one} \\ \text{Complete} \\ \text{Work}}}{}$$

$$\frac{1}{5} \cdot t - \frac{1}{7} \cdot t = 1$$

$LCD = 35$

$7t - 5t = 35$

$2t = 35$ $\boxed{t = 17.5}$

17.5 hrs

⑧ Simplify: $\dfrac{x^{-2}}{1-x^{-2}} = \dfrac{\frac{1}{x^2}}{1-\frac{1}{x^2}} = \dfrac{1}{x^2-1}$

LCD $= x^2$

⑨ John can do a certain job in 8 hrs alone, while Maria can do the same job in 12 hrs alone. How long does it take to do the job if they work together?

$\dfrac{1}{8}\cdot t + \dfrac{1}{12}\cdot t = 1 \Rightarrow \dfrac{t}{8} + \dfrac{t}{12} = 1$

$3t + 2t = 24$
$5t = 24$
$t = 4.8$

LCD $= 24$ 4.8 hrs

⑩ Simplify: $\dfrac{x^2+9x+20}{x^2-25} \div \dfrac{x^2+8x+16}{x^2-9x+20}$

$= \dfrac{(x+4)(x+5)}{(x+5)(x-5)} \cdot \dfrac{(x-4)(x-5)}{(x+4)(x+4)} = \boxed{\dfrac{x-4}{x+4}}$

⑪ Simplify: $\dfrac{4}{3x^2+8x-3} + \dfrac{2}{3x^2-7x+2}$

$= \dfrac{4(x-2)}{(3x-1)(x+3)(x-2)} + \dfrac{2(x+3)}{(3x-1)(x-2)(x+3)}$

$= \dfrac{4(x-2)+2(x+3)}{(3x-1)(x+3)(x-2)} = \dfrac{4x-8+2x+6}{(3x-1)(x+3)(x-2)} = \dfrac{6x-2}{(3x-1)(x+3)(x-2)}$

$= \dfrac{2(3x-1)}{(3x-1)(x+3)(x-2)} = \boxed{\dfrac{2}{(x+3)(x-2)}}$

John traveled 90 miles <u>in the Same time</u> that
Maria traveled 60 miles.
Maria was driving 10 mph slower than John.
find speed for both.

$t_{John} = t_{Maria}$

$\dfrac{\overset{3}{\cancel{\overset{q}{\cancel{90}}}}}{x} = \dfrac{\overset{6}{\cancel{60}}\overset{2}{}}{x-10}$

d =	r	· t
John	90	x	t
Maria	60	$x-10$	t

$3(x-10) = 2x$

$3x - 30 = 2x$

$3x - 2x = 30$

$\boxed{x = 30}$

John 30 mph

Maria 20 mph

John drove 50 miles in the City and
240 miles on the highway. Total time 6 hrs.
His speed on the highway was 10 mph
faster than twice his speed in the City.
find his speed in the City & on the highway.

$t_1 + t_2 = 6$

$\dfrac{50}{x} + \dfrac{240}{2x+10} = 6$

$\dfrac{50}{x} + \dfrac{\cancel{2} \cdot 120}{\cancel{2}(x+5)} = 6$

$\dfrac{\overset{25}{\cancel{50}}}{x} + \dfrac{\overset{60}{\cancel{120}}}{x+5} = \overset{3}{\cancel{6}}$

	d	r	t
City	50	x	t_1
HWY	240	$2x+10$	t_2

Solve $\boxed{\dfrac{25}{x}} + \boxed{\dfrac{60}{x+5}} = \boxed{3}$

$LCD = x(x+5)$

$25(x+5) + 60x = 3x(x+5)$

$$25(x+5) + 60x = 3x(x+5)$$
$$25x + 125 + 60x = 3x^2 + 15x$$
$$85x + 125 = 3x^2 + 15x$$
$$3x^2 + 15x - 85x - 125 = 0$$
$$3x^2 - 70x - 125 = 0$$
$$a = 3 \quad b = -70 \quad c = -125$$
$$b^2 - 4ac = (-70)^2 - 4(3)(-125) = 6400$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{70 \pm \sqrt{6400}}{6} = \frac{70 \pm 80}{6}$$

$x = \frac{70 + 80}{6} = \frac{150}{6}$

$\boxed{x = 25}$

$x = \frac{70 - 80}{6}$

$x = -$

City : 25 mph

Hwy : 60 mph

It takes Larry 9 hrs longer than Linda to do a job.
They can do the job in 6 hrs when they work together.
How long if they work alone.

Linda → x → Rate = $\frac{1}{x}$

Larry → $x+9$ → Rate = $\frac{1}{x+9}$

$$\frac{1}{x} \cdot 6 + \frac{1}{x+9} \cdot 6 = 1$$

$$\frac{6}{x} + \frac{6}{x+9} = 1$$
$$LCD = x(x+9)$$
$$6(x+9) + 6x = x(x+9)$$
$$6x + 54 + 6x = x^2 + 9x$$
$$12x + 54 = x^2 + 9x$$

6

$$12x + 54 = x^2 + 9x$$

$$x^2 + 9x - 12x - 54 = 0$$

$$x^2 - 3x - 54 = 0$$

$$(x + 6)(x - 9) = 0$$

$$x = -6 \qquad x = 9$$

Linda → 9 hrs
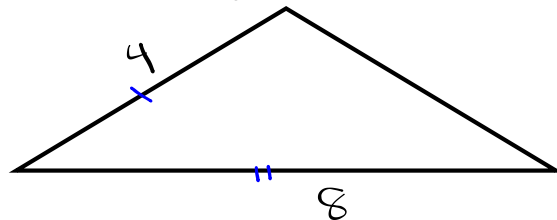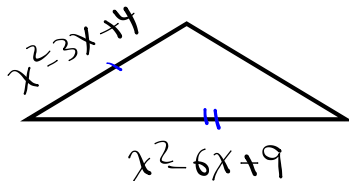
Larry → 18 hrs

SG 20 #12 Similar triangles

$x^2 - 3x + 4$ $x^2 - 6x + 9$ 4 8

$$\frac{x^2 - 3x + 4}{4} = \frac{x^2 - 6x + 9}{8}$$

$$\{\pm 1\}$$

$$2(x^2 - 3x + 4) = 1(x^2 - 6x + 9)$$

$$2x^2 - 6x + 8 = x^2 - 6x + 9$$

$$2x^2 + 8 - x^2 - 9 = 0$$

$$x^2 - 1 = 0$$

$$(x+1)(x-1) = 0$$

$$x = -1 \qquad x = 1$$

7

Final Exam:

1) Due   SG 20

2) Review   exams 1, 2, and 3

3) Review   recent SGs & Project

4) Make Sure to have a Calc.